ON THE SOLUTION OF A LINEAR PARTIALLY INTEGRAL EQUATION IN THE SPACE $L_2([a, b]^3)$.

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Abstract: The study of spectral properties, in particular bound states, of multiparticle operators of quantum mechanics and solid state physics is closely related to the problem of solving integral equations with partial integrals (partial integral equations) for functions of three variables. In this article, we study the question of the existence and uniqueness of the solution of a linear partial integral equation for functions of three variables with degenerate kernels and many parameters in a complex Hilbert space. It is proved that under natural conditions, equation (1) has a unique solution, which is expressed through the data and their integrals of the considered equation. The general view of the solution is found. When solving the partial integral equation under study, the Fredholm method was developed for solving a linear integral equation of the second kind with a parameter.

Key words: partial integral equation, function, Fredholm integral equation, kernel, parameter, condition, system of equations, Fredholm method, Fredholm determinant, unique solution, expression, formula.

INTRODUCTION

Functional equations have occupied an important place in the work of mathematicians for a long time. Recently, the attention of mathematicians has been especially directed to a special type of functional equations, the so-called integral equations, i.e. such equations in which the unknown function appears under the integral sign. The solution of an equation of this kind is sometimes interpreted as the inverse of a definite integral. The study of spectral properties, in particular bound states, of many-particle operators of quantum mechanics and solid state physics is closely related to the problem of solving partial-integral equations for functions of three variables. For example, the problem for the equation of eigenfunctions of the four-particle Schrödinger operator with pair interactions [1] leads to the solution of integral equations with partial integrals for functions of three variables. The question of the existence of a solution to an integral equation with partial integrals for functions of two variables was considered in the works of Abdus Salam [2] and others, and for functions of three variables in [3-6], and in connection with the study of the spectral properties of many-particle operators was studied in books by S.P. Merkuriev, L.D. Faddeev [7], S. Albeverio., F. Gestesi, Z. Heeg-Kron, H. Holden [8] and in the works of S.N. Lakaev [9-10]. The aim of this work is to study the solvability of partially integral equations for functions of three variables in the complex Hilbert space $L_2([a, b]^3)$.

MATERIALS AND METHODS

Research methodology. In the proof of the existence and uniqueness of the solution of a partial integral equation with degenerate kernels and many parameters for functions of three variables, methods of the theory of integral equations, methods of the theory of functions and functional analysis are used.

Problem statement and formulation of the main result. In a complex Hilbert space $L_2(D), D=[a,b]\times[a,b]\times[a,b]$ consider a linear partial integral equation for functions of three variables with degenerate kernels and many parameters of the form

$$\varphi(x,y,z)= f(x,y,z)+\lambda\int_a^b \psi_1(x)\overline{\psi_1(t)}\varphi(t,y,z)dt+$$

$$+\mu\int_a^b \psi_2(x)\overline{\psi_2(t)}\varphi(x,t,z)dt+\nu\int_a^b \psi_3(x)\overline{\psi_3(t)}\varphi(x,y,t)dt \qquad (1)$$

and the corresponding homogeneous equation

$$\varphi(x,y,z)=\lambda\int_a^b \psi_1(x)\overline{\psi_1(t)}\varphi(t,y,z)dt+$$

$$+\mu\int_a^b \psi_2(x)\overline{\psi_2(t)}\varphi(x,t,z)dt+\nu\int_a^b \psi_3(x)\overline{\psi_3(t)}\varphi(x,y,t)dt. \qquad (2)$$

Here functions $\psi_i, i=1,2,3$ belong to space $L_2([a,b])$ and satisfy the conditions: $(\psi_i,\psi_j)=\delta_{ij}$, $i,j=1,2,3;$ free function f is an element of $L_2(D)$, λ,μ and ν numeric parameters, φ - the required function from $L_2(D)$.

Theorem. Let $\lambda\neq 1, \mu\neq 1, \nu\neq 1, \lambda+\mu\neq 1, \lambda+\nu\neq 1, \mu+\nu\neq 1$ and $\lambda+\mu+\nu\neq 1$. Then: a) equation (1) for any f has a unique solution expressed by the formula

$$\varphi(x,y,z)=f(x,y,z)+\frac{\lambda\psi_1(x)}{1-\lambda}\int_a^b\overline{\psi_1(t)}f(t,y,z)dt+$$

$$+\frac{\mu\psi_2(y)}{1-\mu}\int_a^b\overline{\psi_2(s)}f(x,s,z)ds+\frac{\nu\psi_3(z)}{1-\nu}\int_a^b\overline{\psi_3(u)}f(x,y,u)du+$$

$$+\frac{\lambda\mu(2-\lambda-\mu)\psi_1(x)\psi_2(y)}{(1-\lambda)(1-\mu)(1-\lambda-\mu)}\int_a^b\int_a^b\overline{\psi_1(t)}\overline{\psi_2(s)}f(t,s,z)dtds+$$

$$+\frac{\lambda\nu(2-\lambda-\nu)\psi_1(x)\psi_3(z)}{(1-\lambda)(1-\nu)(1-\lambda-\nu)}\int_a^b\int_a^b\overline{\psi_1(t)}\overline{\psi_3(u)}f(t,y,u)dtdu+$$

$$+\frac{\mu\nu(2-\mu-\nu)\psi_2(y)\psi_3(z)}{(1-\mu)(1-\nu)(1-\mu-\nu)}\int_a^b\int_a^b\overline{\psi_2(s)}\overline{\psi_3(u)}f(x,s,u)dsdu+$$

$$+\frac{C\psi_1(x)\psi_2(y)\psi_3(z)}{(1-\lambda)(1-\mu)(1-\nu)(1-\lambda-\mu)(1-\lambda-\nu)(1-\mu-\nu)(1-\lambda-\mu-\nu)}\times$$

$$\times\int_a^b\int_a^b\int_a^b\overline{\psi_1(t)}\overline{\psi_2(s)}\overline{\psi_3(u)}f(t,s,u)dtdsdu, \qquad (1.1)$$

Where C - is expressed through λ,μ and ν;

b) The homogeneous equation (2) has only a trivial solution.

Proof of the theorem. Let us introduce the following notation:

$$\begin{cases}\alpha_1(y,z)=\int_a^b\overline{\psi_1(t)}\varphi(t,y,z)dt\\ \alpha_2(x,z)=\int_a^b\overline{\psi_2(t)}\varphi(x,t,z)dt\\ \alpha_3(x,y)=\int_a^b\overline{\psi_3(t)}\varphi(x,y,t)dt\end{cases} \qquad (3)$$

Then equation (1) takes the form

$$\varphi(x,y,z) = f(x,y,z) + \lambda\psi_1(x)\alpha_1(y,z) + \mu\psi_2(y)\alpha_2(x,z) + \nu\psi_3(z)\alpha_3(x,y). \qquad (4)$$

Substitute expression (4) into system (3):

$$\begin{cases} \alpha_1(y,z) = \int\limits_a^b \overline{\psi_1(t)}[f(t,y,z) + \lambda\psi_1(t)\alpha_1(y,z) + \mu\psi_2(y)\alpha_2(t,z) + \nu\psi_3(z)\alpha_3(t,y)]dt \\ \alpha_2(x,z) = \int\limits_a^b \overline{\psi_2(t)}[f(x,t,z) + \lambda\psi_1(x)\alpha_1(t,z) + \mu\psi_2(t)\alpha_2(x,z) + \nu\psi_3(z)\alpha_3(x,t)]dt \\ \alpha_3(x,y) = \int\limits_a^b \overline{\psi_3(t)}[f(x,y,t) + \lambda\psi_1(x)\alpha_1(y,t) + \mu\psi_2(y)\alpha_2(x,t) + \nu\psi_3(t)\alpha_3(x,y)]dt \end{cases} . \qquad (5)$$

To solve system (5), we first give an idea for a solution. It is enough to obtain some Fredholm integral equation of the second kind, with respect to some $\alpha_i, i=1,2,3$, for example, with respect to α_3. For this purpose, it is sufficient to solve the partially integral equation for $\alpha_3(x,y)$, i.e. the function must be expressed through the integrals *f* and $\psi_i, i=1,2,3$. Partially we arrive at the integral equation α_3 with respect to the following way: $\alpha_1(y,z)$ and $\alpha_2(x,z)$ express through $\alpha_3(x,y)$, and for this we find α_1 from the first equation of system (5) and substitute it into the second equation. As a result, we obtain a partially integral equation for α_2. Solving the resulting equation, we find α_2, i.e. we express it through the integrals *f*, $\psi_i, i=1,2,3$ and α_3. Then, using the found expression α_2, we find the function α_1 which is also expressed through the integrals *f*, $\psi_i, i=1,2,3$ and α_3. Further, substituting the found expressions α_1 and α_2 into the third equation of system (5), we obtain the required partial integral equation with respect to α_3. And the last equation is reduced to the solution of some Fredholm integral equation of the second kind with a degenerate kernel. It is known that the Fredholm integral equation of the second kind can be solved explicitly under certain conditions. Thus, we define α_3 through the integrals *f*, $\psi_i, i=1,2,3$. Therefore, we obtain expressions for α_1 and α_2 through the integrals *f*, $\psi_i, i=1,2,3$, finally.

Now let's start solving system (5). From the first equation of system (5) we find $\alpha_1(y,z)$:

$$\alpha_1(y,z)\left[1-\lambda\int_a^b \overline{\psi_1(t)}\psi_1(t)dt\right]=\int_a^b \overline{\psi_1(t)}f(t,y,z)dt+$$

$$+\mu\psi_2(y)\int_a^b \overline{\psi_1(t)}\alpha_2(t,z)+\nu\psi_3(z)\int_a^b \overline{\psi_1(t)}\alpha_3(t,y)]dt\,.$$

Here we note that $(\psi_1,\psi_1)=\int_a^b \overline{\psi_1(t)}\psi_1(t)dt=1$. That's why if $\lambda\neq 1$,then

$$\alpha_1(y,z)=\frac{1}{1-\lambda}\int_a^b \overline{\psi_1(t)}f(t,y,z)dt+$$

$$+\frac{\mu\psi_2(y)}{1-\lambda}\int_a^b \overline{\psi_1(t)}\alpha_2(t,z)dt+\frac{\nu\psi_3(z)}{1-\lambda}\int_a^b \overline{\psi_1(t)}\alpha_3(t,y)]dt\quad . \tag{6}$$

Substituting expression (6) into the second equation of system (5), with $\mu\neq 1$ taking into account the condition, we arrive at $(\psi_2,\psi_2)=1$ a partially integral equation for the following α_2 form:

$$\alpha_2(x,z)=F(f,\alpha_3;x,z)=\frac{\lambda\mu}{(1-\lambda)(1-\mu)}\int_a^b \overline{\psi_2(t)}\alpha_2(t,z)dt,\qquad \text{где} \tag{7}$$

$$F(f,\alpha_3;x,z)=\frac{1}{1-\mu}\int_a^b \overline{\psi_2(s)}f(x,s,z)ds+\frac{\lambda\mu\psi_1(x)}{(1-\lambda)(1-\mu)}\int_a^b\int_a^b \overline{\psi_2(t)}\,\overline{\psi_2(s)}f(t,s,z)dtds+$$

$$+\frac{\nu\psi_3(z)}{1-\mu}\int_a^b \overline{\psi_2(s)}\alpha_3(x,s)ds+\frac{\lambda\nu\psi_1(x)\psi_3(z)}{(1-\lambda)(1-\mu)}\int_a^b\int_a^b \overline{\psi_2(t)}\,\overline{\psi_2(s)}\alpha_3(t,s)dtds\;. \tag{8}$$

We solve equation (7). $D(\lambda,\mu)=\dfrac{1-\lambda-\mu}{(1-\lambda)(1-\mu)}$ is the Fredholm determinant of equation (7). Therefore, if $\lambda+\mu\neq 1$, then partially integral equation (7) is the only solution for any $F(f,\alpha_3;x,z)$ form

$$\alpha_2(x,z)=F(f,\alpha_3;x,z)+\frac{\lambda\mu\psi_1(x)}{1-\lambda-\mu}\int_a^b \overline{\psi_1(t)}F(f,\alpha_3;t,z)dt,$$

For almost all $z\in[a,b]$, taking into account the expression (8) of the function *F*, we finally obtain the expression for α_2 in the form

$$\alpha_2(x,z)=\frac{1}{1-\mu}\int_a^b \overline{\psi_2(s)} f(x,s,z)ds+\frac{\lambda\psi_1(x)}{(1-\lambda)(1-\lambda-\mu)}\int_a^b\int_a^b \overline{\psi_1(t)}\,\overline{\psi_2(s)} f(t,s,z)dtds+$$

$$+\frac{\nu\psi_3(z)}{1-\mu}\int_a^b \overline{\psi_2(s)}\alpha_3(x,s)ds+\frac{\lambda\nu\psi_3(z)}{(1-\lambda)(1-\lambda-\mu)}\int_a^b\int_a^b \overline{\psi_1(t)}\,\overline{\psi_2(s)}\alpha_3(t,s)dtds. \qquad (9)$$

Using (9) instead of (6), we write the following:

$$\alpha_1(y,z)=\frac{1}{1-\lambda}\int_a^b \overline{\psi_1(t)} f(t,y,z)dt+\frac{\mu\psi_2(y)}{(1-\lambda)(1-\lambda-\mu)}\int_a^b\int_a^b \overline{\psi_1(t)}\,\overline{\psi_2(s)} f(t,s,z)dtds$$

$$+\frac{\nu\psi_3(z)}{1-\mu}\int_a^b \overline{\psi_1(s)}\alpha_3(t,y)dt++\frac{\mu\nu\psi_2(y)\psi_3(z)}{(1-\lambda)(1-\lambda-\mu)}\int_a^b\int_a^b \overline{\psi_1(t)}\,\overline{\psi_2(s)}\alpha_3(t,s)dtds. \qquad (10)$$

Further, the found expressions of the functions α_1 and α_2 from (10) and (9) are substituted into the third equation of system (5) and taking into account the condition $(\psi_3,\psi_3)=1$, at $\nu\neq 1$ we arrive at a partially integral equation with respect to α_3:

$$\alpha_3(x,y)=F_1(f;x,z)+\frac{\lambda\nu\psi_1(x)}{(1-\lambda)(1-\mu)}\int_a^b \overline{\psi_1(t)}\alpha_3(t,y)dt+\frac{\mu\nu\psi_2(y)}{(1-\lambda)(1-\nu)}\int_a^b \overline{\psi_2(t)}\alpha_3(x,s)ds+$$

$$+\frac{\lambda\mu\nu(2-\lambda-\mu)\psi_1(x)\psi_2(y)}{(1-\lambda)(1-\mu)(1-\lambda-\mu)}\int_a^b\int_a^b \overline{\psi_1(t)}\,\overline{\psi_2(s)}\alpha_3(t,s)dtds\ . \qquad (11)$$

Here

$$F_1(f;x,z)=\frac{1}{1-\nu}\int_a^b \overline{\psi_3(u)} f(x,y,u)du+\frac{\lambda\psi_1(x)}{(1-\lambda)(1-\nu)}\int_a^b\int_a^b \overline{\psi_1(t)}\,\overline{\psi_3(u)} f(t,y,u)dtdu+$$

$$+\frac{\mu\psi_2(y)}{(1-\mu)(1-\nu)}\int_a^b\int_a^b \overline{\psi_2(s)}\,\overline{\psi_3(u)} f(x,s,u)dsdu+$$

$$+\frac{\lambda\mu\nu(2-\lambda-\mu)\psi_1(x)\psi_2(y)}{(1-\lambda)(1-\mu)(1-\lambda-\mu)}\int_a^b\int_a^b\int_a^b \overline{\psi_1(t)}\,\overline{\psi_2(s)}\,\overline{\psi_3(u)} f(t,s,u)dtdsdu.$$

Assuming that

$$\hat{F}(f,\alpha_3;x,y)=F_1(f;x,z)+\frac{\mu\nu\psi_2(y)}{(1-\lambda)(1-\nu)}\int_a^b \overline{\psi_2(t)}\alpha_3(x,s)ds+$$

$$+\frac{\lambda\mu\nu(2-\lambda-\mu)\psi_1(x)\psi_2(y)}{(1-\lambda)(1-\mu)(1-\lambda-\mu)}\int_a^b\int_a^b\overline{\psi_1(t)}\,\overline{\psi_2(s)}\alpha_3(t,s)dtds,$$

The equation (11) can be reduced to the form

$$\alpha_3(x,y)=\hat{F}(f,\alpha_3;x,y)+\frac{\lambda\nu\psi_1(x)}{(1-\lambda)(1-\mu)}\int_a^b\overline{\psi_1(t)}\alpha_3(t,y)dt. \quad (12)$$

The Fredholm determinant of equation (12) has the form $D(\lambda,\nu)=\frac{1-\lambda-\nu}{(1-\lambda)(1-\nu)}$; therefore, if $1-\lambda-\nu\neq 0$, then partially integral equation (12) has a unique solution of the form

$$\alpha_3(x,y)=\hat{F}(f,\alpha_3;x,y)+\frac{\lambda\nu\psi_1(x)}{1-\lambda-\nu}\int_a^b\overline{\psi_1(t)}\hat{F}(f,\alpha_3;t,y)dt.$$

Given the expression $\hat{F}$ in the latter, we have:

$$\alpha_3(x,y)=G(f,\alpha_3;x,y)+\frac{\mu\nu\psi_2(y)}{(1-\mu)(1-\nu)}\int_a^b\overline{\psi_2(s)}\alpha_3(x,s)ds, \quad (13)$$

In which

$$G(f,\alpha_3;x,y)=\frac{1}{1-\nu}\int_a^b\overline{\psi_3(u)}f(x,y,u)du+\frac{\lambda\psi_1(x)}{(1-\lambda)(1-\lambda-\nu)}\int_a^b\int_a^b\overline{\psi_1(t)}\,\overline{\psi_3(u)}f(t,y,u)dtdu+$$

$$+\frac{\mu\psi_2(y)}{(1-\mu)(1-\nu)}\int_a^b\int_a^b\overline{\psi_2(s)}\,\overline{\psi_3(u)}f(x,s,u)dsdu+$$

$$+\frac{\lambda\mu(2-\lambda-\mu)\psi_1(x)\psi_2(y)}{(1-\lambda-\mu)(1-\lambda-\nu)(1-\mu)(1-\nu)}\int_a^b\int_a^b\int_a^b\overline{\psi_1(t)}\,\overline{\psi_2(s)}\,\overline{\psi_3(u)}f(t,s,u)dtdsdu+$$

$$+\frac{\lambda\mu(2-\lambda-\mu)\psi_1(x)\psi_2(y)}{(1-\lambda-\mu)(1-\lambda-\nu)(1-\mu)(1-\nu)}\int_a^b\int_a^b\overline{\psi_1(t)}\,\overline{\psi_2(s)}\alpha_3(t,s)dtds\ . \quad (14)$$

Note that equation (13) is an analogue of equation (12). Therefore, its determinant has the form $D(\mu,\nu)=\frac{1-\mu-\nu}{(1-\mu)(1-\nu)}$ and, if $1-\mu-\nu\neq 0$, then the equation has a unique solution for any function of the form $G(f,\alpha_3;x,y)$

$$\alpha_3(x,y)=G(f,\alpha_3;x,y)+\frac{\mu\nu\psi_2(y)}{1-\mu-\nu}\int_a^b\overline{\psi_2(s)}G(f,\alpha_3;x,s)ds$$

for every fixed value of the variable *x*. Taking into account (14), we arrive at the complete Fredholm integral equation of the second kind with respect to α_3:

$$\alpha_3(x,y) = \hat{G}(f;x,y) + \frac{\lambda\mu\nu(2-\lambda-\mu-\nu)\psi_1(x)\psi_2(y)}{(1-\lambda-\mu)(1-\lambda-\nu)(1-\mu-\nu)} \int_a^b\int_a^b \overline{\psi_1(t)}\,\overline{\psi_2(s)}\alpha_3(t,s)dtds. \qquad (15)$$

Here

$$\hat{G}(f;x,y) = \frac{1}{1-\nu}\int_a^b \overline{\psi_3(u)} f(x,y,u)du + \frac{\lambda\psi_1(x)}{(1-\lambda)(1-\lambda-\nu)}\int_a^b\int_a^b \overline{\psi_1(t)}\,\overline{\psi_3(u)} f(t,y,u)dtdu +$$

$$+\frac{\mu\psi_2(y)}{(1-\mu)(1-\nu)}\int_a^b\int_a^b \overline{\psi_2(s)}\,\overline{\psi_3(u)} f(x,s,u)dsdu +$$

$$+\frac{\lambda\mu[2-\lambda-\mu-\nu(2-\nu)]\psi_1(x)\psi_2(y)}{(1-\lambda-\mu)(1-\lambda-\nu)(1-\mu-\nu)(1-\nu)}\int_a^b\int_a^b\int_a^b \overline{\psi_1(t)}\,\overline{\psi_2(s)}\,\overline{\psi_3(u)} f(t,s,u)dtdsdu. \qquad (16)$$

The Fredholm determinant corresponding to the kernel of the integral equation (15) has the form: $D(\lambda,\mu,\nu) = \dfrac{(1-\lambda)(1-\mu)(1-\lambda-\mu-\nu)}{(1-\lambda-\mu)(1-\lambda-\nu)(1-\lambda-\nu)}$. As $\lambda \neq 1,\ \mu \neq 1,\ \nu \neq 1$, then if $1-\lambda-\mu-\nu \neq 0$, i.e. $D(\lambda,\mu,\nu) \neq 0$, then the integral equation has a unique solution expressed by the formula

$$\alpha_3(x,y) = \hat{G}(f;x,y) + \frac{\lambda\mu\nu(2-\lambda-\mu-\nu)\psi_1(x)\psi_2(y)}{(1-\lambda)(1-\mu)(1-\nu)(1-\lambda-\mu-\nu)}\int_a^b\int_a^b \overline{\psi_1(t)}\,\overline{\psi_2(s)}\hat{G}(f;t,s)dtds.$$

Now, taking into account the expression of the function $\hat{G}$ from (16), we finally get $\alpha_3(x,y)$ in the form

$$\alpha_3(y,z) = \frac{1}{1-\nu}\int_a^b \overline{\psi_3(u)} f(x,y,u)du + \frac{\lambda\psi_1(x)}{(1-\lambda)(1-\lambda-\nu)}\int_a^b\int_a^b \overline{\psi_1(t)}\,\overline{\psi_3(u)} f(t,y,u)dtdu +$$

$$+C_{12}\psi_1(x)\psi_2(y)\int_a^b\int_a^b\int_a^b \overline{\psi_1(t)}\,\overline{\psi_2(s)}\,\overline{\psi_3(u)} f(t,s,u)dtdsdu, \qquad (17)$$

Where

$$C_{12} = \frac{\lambda\mu\nu\{(2-\lambda-\mu-\nu)(1-\lambda-\mu)[1-\mu(1-\mu)-\nu]-(2-\nu)\}}{(1-\lambda)(1-\nu)(1-\lambda-\mu)(1-\lambda-\nu)(1-\mu-\nu)(1-\lambda-\mu-\nu)} +$$

$$+\frac{\lambda\mu[1-\lambda-\mu-\nu)(2-\nu)]}{(1-\nu)(1-\lambda-\mu)(1-\lambda-\nu)(1-\mu-\nu)}.$$

Further, substituting (17) into (9) and (10), we have:

$$\alpha_2(x,z)=\frac{1}{1-\mu}\int_a^b\overline{\psi_2(s)}f(x,s,z)ds+\frac{\lambda\psi_1(x)}{(1-\lambda)(1-\lambda-\mu)}\int_a^b\int_a^b\overline{\psi_1(t)}\overline{\psi_2(s)}f(t,s,z)dtds+$$

$$+\frac{\nu\psi_3(z)}{(1-\mu)(1-\lambda-\mu)}\int_a^b\int_a^b\overline{\psi_2(s)}\overline{\psi_3(u)}f(x,s,u)dtds+$$

$$+C_{13}\psi_1(x)\psi_3(z)\int_a^b\int_a^b\int_a^b\overline{\psi_1(t)}\overline{\psi_2(s)}\overline{\psi_3(u)}f(t,s,u)dtdsdu\ , \qquad (18)$$

Where $C_{13}=\dfrac{\nu[C_{12}(1-\nu)(1-\mu)(1-\lambda-\nu)+\lambda(2-\lambda-\nu)]}{(1-\lambda-\mu)(1-\mu-\nu)(1-\nu)(1-\mu)}$;

$$\alpha_1(y,z)=\frac{1}{1-\lambda}\int_a^b\overline{\psi_1(t)}f(t,y,z)dt+\frac{\mu\psi_2(y)}{(1-\lambda)(1-\lambda-\mu)}\int_a^b\int_a^b\overline{\psi_1(t)}\overline{\psi_2(s)}f(t,s,z)dtds+$$

$$+\frac{\mu\psi_3(z)}{(1-\lambda)(1-\lambda-\nu)}\int_a^b\int_a^b\overline{\psi_1(t)}\overline{\psi_3(u)}f(t,y,u)dtdu+ \qquad (19)$$

$$+C_{23}\psi_2(y)\psi_3(z)\int_a^b\int_a^b\int_a^b\overline{\psi_1(t)}\overline{\psi_2(s)}\overline{\psi_3(u)}f(t,s,u)dtdsdu,$$ in which

$$C_{23}=\frac{\nu[C_{12}(1-\mu-\nu)(1-\lambda-2\mu+\mu\nu)+\mu(2-\lambda-\mu-\nu)+\lambda(1-\lambda-\mu)]}{(1-\mu)(1-\nu)(1-\lambda-\mu)(1-\mu-\nu)}.$$

Thus, in expressions (19), (18) and (17), respectively, functions α_1, α_2 and α_3,the solution of system (5) is determined. Substituting these expressions in (4), we obtain a solution to equation (1) expressed by formula (1.1). From the uniqueness, α_1, α_2 and α_3 the uniqueness of the function of equation (1) follows. Part a) of the theorem is proved. The proof of part b) follows from the proof of part a).

CONCLUSION

Partially integral equations for functions of three variables have not been studied well enough earlier. In a complex Hilbert space $L_2([a,b]^3)$, the solvability of a partial integral equation for functions of three variables with degenerate kernels and many parameters and the corresponding homogeneous equation is studied. In the following papers, we present the results of a complete study of the homogeneous equation (2) for all possible values of the parameters and the description of the solution space, as well as the necessary and sufficient conditions for the existence of a solution to equations (1) and (2).

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